Section D

7. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4). **7.5**

(b) Prove that :

$$\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + 3xz^2\hat{k}$$

is irrotational. **7.5**

8. (a) Evaluate $\iint_S \vec{A}.\hat{n}\, dS$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2 z\hat{k}$ and S is the curved surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. **7.5**

(b) Use Green's theorem in a plane to evaluate the integral $\oint_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$, where C is the boundary in the *xy*-plane of the area enclosed by the *x*-axis and the semi-circle $x^2 + y^2 = 1$ in the upper half *xy*-plane. **7.5**

Sep-21-00002

B. Tech. EXAMINATION, 2021

Semester I (CBCS)

ENGINEERING MATHEMATICS–I (A & B)

MA-101

Time : 2 *Hours* *Maximum Marks* : 60

The candidates shall limit their answers precisely within 20 pages only (A4 size sheets/assignment sheets), no extra sheet allowed. The candidates should write only on one side of the page and the back side of the page should remain blank. Only blue ball pen is admissible.

Note : Attempt *Four* questions in all, selecting *one* question from any of the Sections A, B, C and D. Q. No. **9** is compulsory.

Section A

1. (a) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$. **7.5**

(b) Show that the vectors :

$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$ and $X_4 = (-3, 7, 2)$ are linearly dependent and find a relation between them. **7.5**

2. (a) Verify Cayley-Hamilton Theorem for the following matrix and hence find A^{-1}, when

$$A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & -2 \end{bmatrix}.$$ **7.5**

(b) Prove that the matrices A and P^{-1} AP have the same eigen values, P being an invertible matrix of same order as A. **7.5**

Section B

3. (a) Prove that $\sin(\alpha + n\theta) - e^{i\alpha}\sin n\theta = e^{-in\theta}\sin\alpha$. **7.5**

(b) If $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, then prove that : **7.5**

(i) $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$

(ii) $\cosh u = \sec\theta$.

4. (a) Find all values of z such that $\sin z = 4$. **7.5**

(b) If $f(z)$ is a holomorphic function of z, show that :

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2.$$ **7.5**

Section C

5. (a) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}.$$ **7.5**

(b) If $z = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find the value of $\frac{dz}{dx}$, when $x = y = a$. **7.5**

6. (a) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ by changing into polar co-ordinates. **7.5**

(b) Prove that $\int_0^1 x^5(1-x^3)^{10}\,dx = \frac{1}{396}$. **7.5**

(Compulsory Question)

9. (a) Write the definition of linearly dependent and linearly independent vectors.

(b) Prove that sin z is not bounded, where $z = x + iy$ is a complex variable.

(c) Find the general value of Log (–3).

(d) Show that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$.

(e) Expand a^x, $a > 0$ in terms of Maclaurin's series.

(f) Evaluate the double integral $\int_1^a \int_1^b \frac{dydx}{xy}$.

(g) Prove that $\Gamma(n) = (n-1)\Gamma(n-1)$.

(h) Prove that if $\vec{F}(t)$ has a constant direction, then $\vec{F} \times \frac{d\vec{F}}{dt} = \vec{0}$.

(i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that grad $r = \frac{\vec{r}}{r}$.

(j) Evaluate $\iint_C \vec{r}.\vec{n}\,dS$, where S is the closed surface. **10×1.5=15**